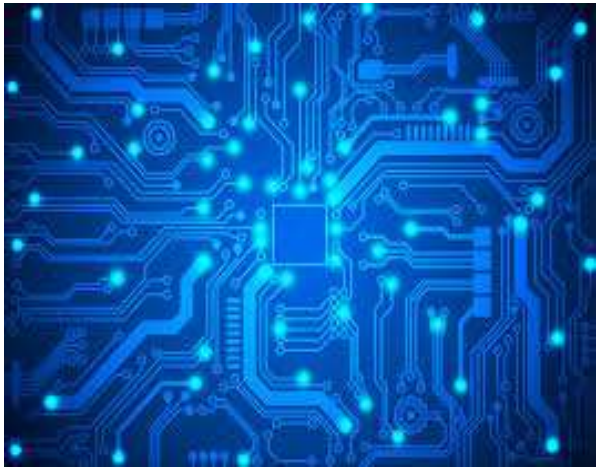
Digital Systems: Boolean Algebra and Logic Gates

VINOD KUMAR VERMA, PGT(CS), KV OEF KANPUR &
SACHIN BHARDWAJ, PGT(CS), KV NO.1 TEZPUR



DEFINITION:

Boolean Algebra is the algebra of truth values and operations performing on them which is used in Digital Circuits for performing logical operations.

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Boolean Constants and Variables

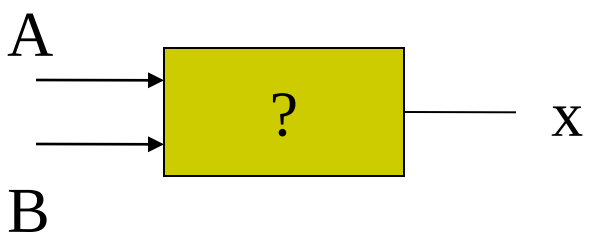
- Logical statements can have either two values yes or no, true or false, 0 or 1.
- Boolean 0 and 1 do not represent actual numbers but instead represent the state, or logic level.

Logic 0	Logic 1
False	True
Off	On
Low	High
No	Yes
Open switch	Closed switch

Truth Tables

- A truth table is a means for describing how a logic circuit's output depends on the logic levels present at the circuit's inputs.

Inputs		Output
A	B	x
1	1	1
0	1	0
1	0	0
0	0	0



Three Basic Logic Operators

- OR
- AND
- NOT

OR Operation

- Boolean expression for the OR operation:

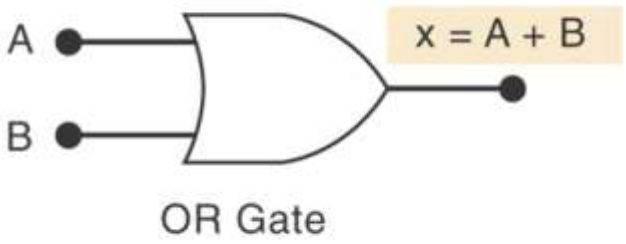
$$x = A + B$$

- The above expression is read as "x equals A OR B"

OR

A	B	x = A + B
0	0	0
0	1	1
1	0	1
1	1	1

(a)



(b)

AND Operation

- Boolean expression for the AND operation:

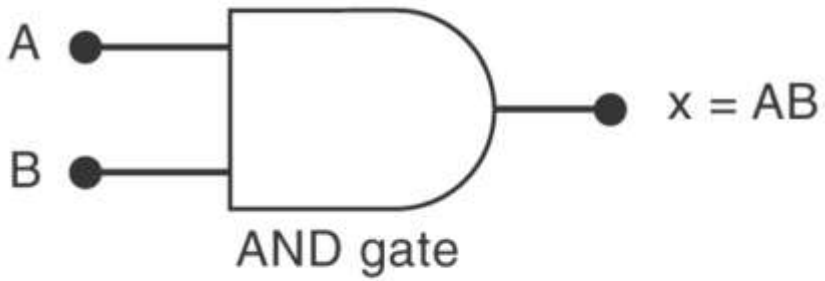
$$x = A \cdot B$$

- The above expression is read as "x equals A AND B"

AND

A	B	x = A · B
0	0	0
0	1	0
1	0	0
1	1	1

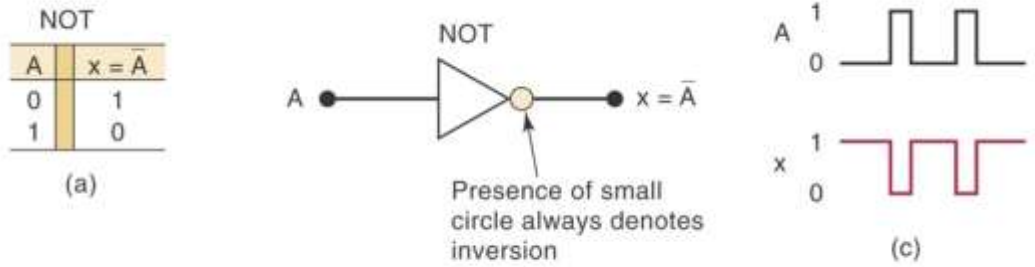
(a)



(b)

NOT Operation

- The NOT operation is an unary operation, taking only one input variable.
- Boolean expression for the NOT operation:
  $x = \overline{A}$
- The above expression is read as "x equals the inverse of A"
- Also known as inversion or complementation.
- Can also be expressed as: A'

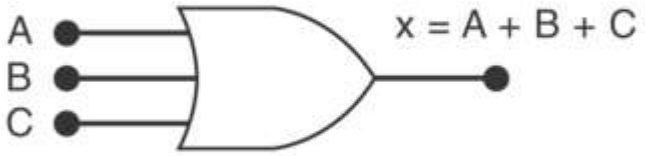


LOGIC GATES

- OR
- AND
- NOT
- NAND
- NOR

OR Gate

- An OR gate is a gate that has two or more inputs and whose output is equal to the OR combination of the inputs.

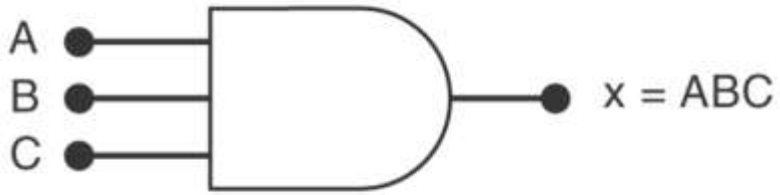


A	B	C	x = A + B + C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

AND Gate

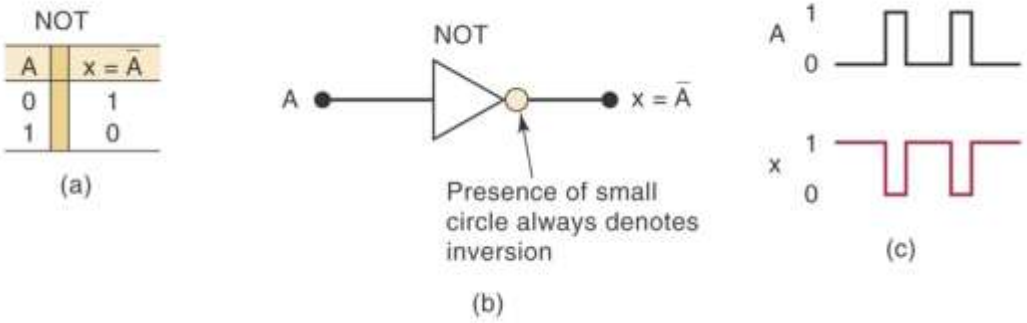
- An AND gate is a gate that has two or more inputs and whose output is equal to the AND product of the inputs.

A	B	C	x = ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



NOT Gate

The symbol and Truth Table for NOT gate is given below:

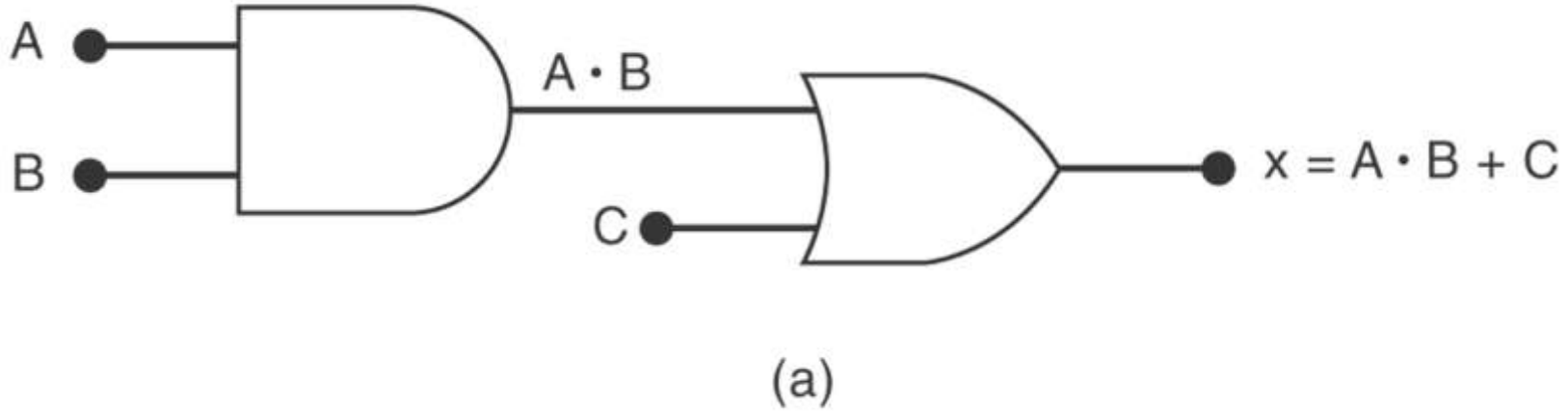


A

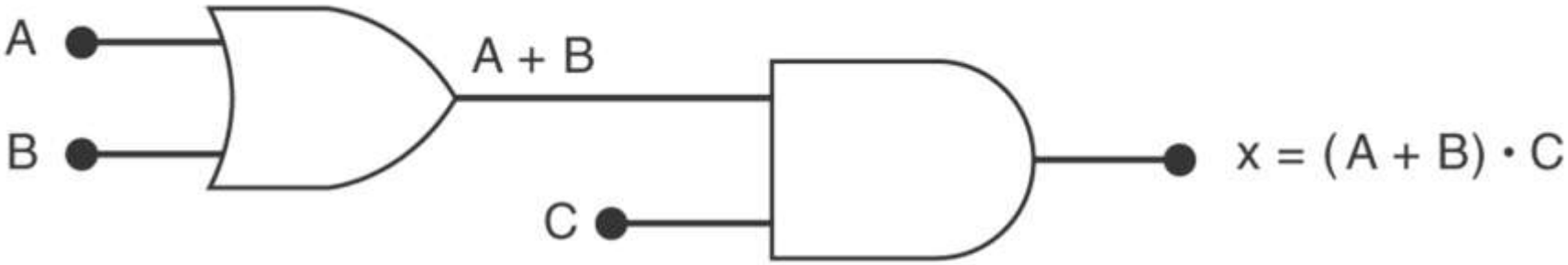
Describing Logic Circuits Algebraically

- Any logic circuits can be built from the three basic building blocks: OR, AND, NOT
- Example 1: $x = A\ B + C$
- Example 2: $x = (A+B)C$
- Example 3: $x = \overline{(A+B)}$

Examples 1,2

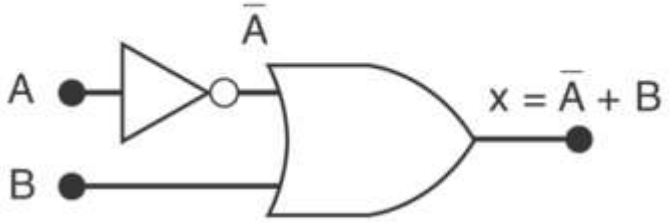


(a)

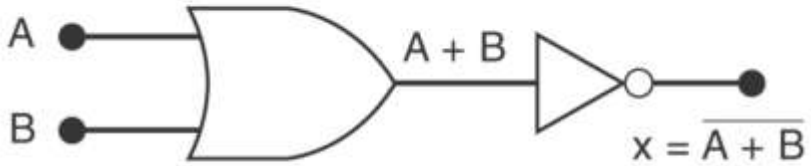


(b)

Examples 3



(a)



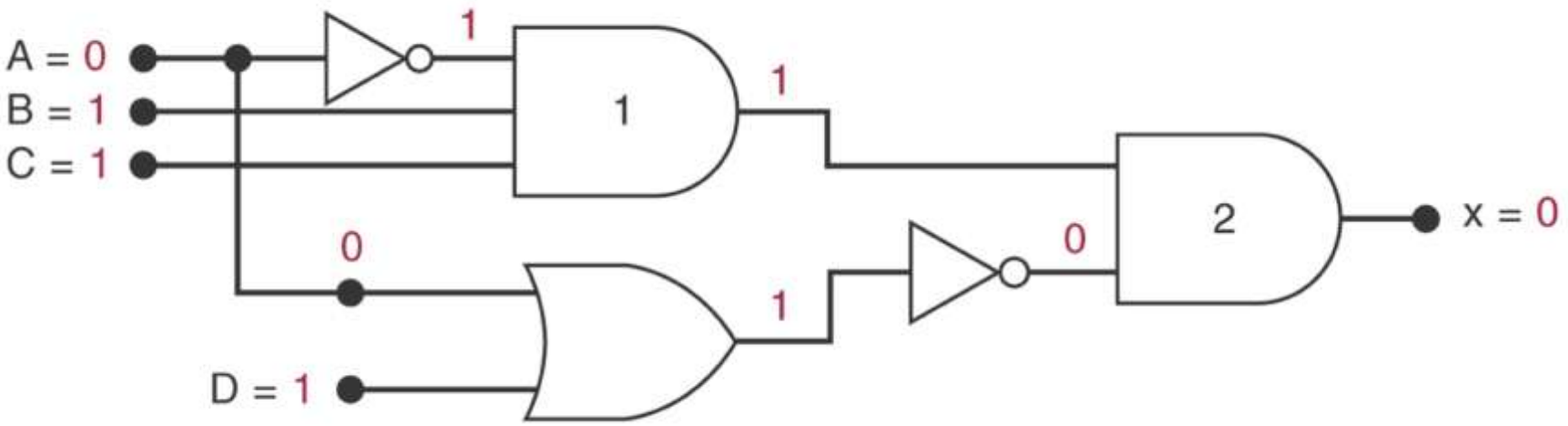
(b)

Evaluating Logic-Circuit Outputs

- $x = \overline{ABC(A+D)}$

- Determine the output x given A=0, B=1, C=1, D=1.
- Can also determine output level from a diagram

Examples



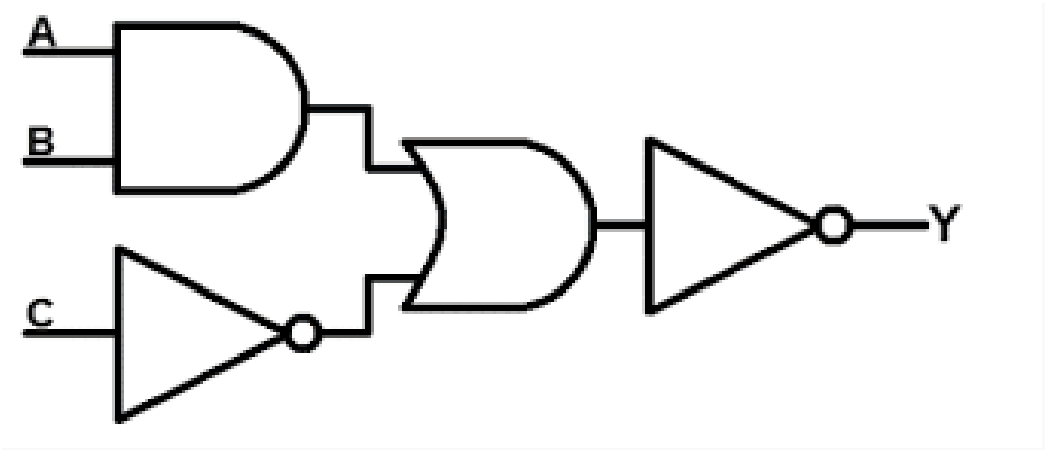
Draw the Logic Gate:

- AB+C'D
- A(B+C') + B'D'
- A' [(B+C)' + AB]

Draw the Truth Table:

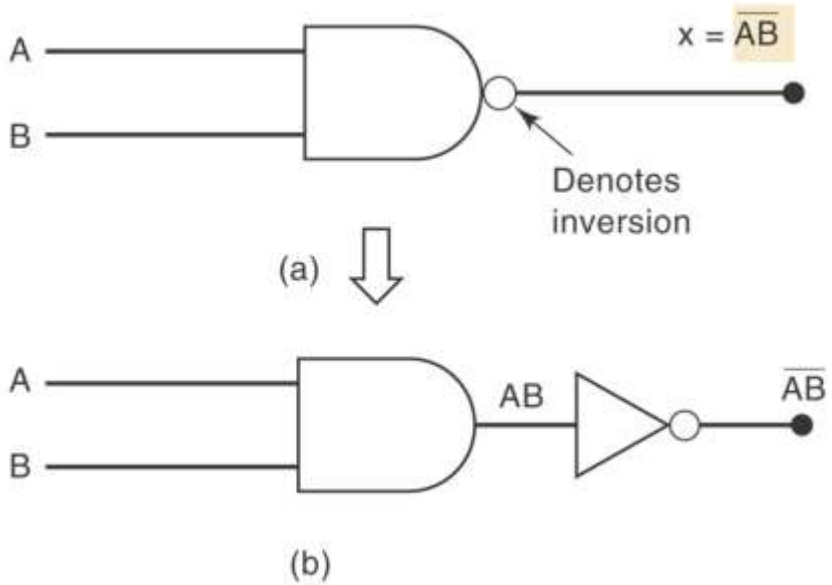
- AB+(BC)'
- A(B'+C') + BC'
- X'[(Y+Z)' + XY]

Derive the Boolean Expression



NAND Gate

- Boolean expression for the NAND operation:
  $x = \overline{A\ B}$

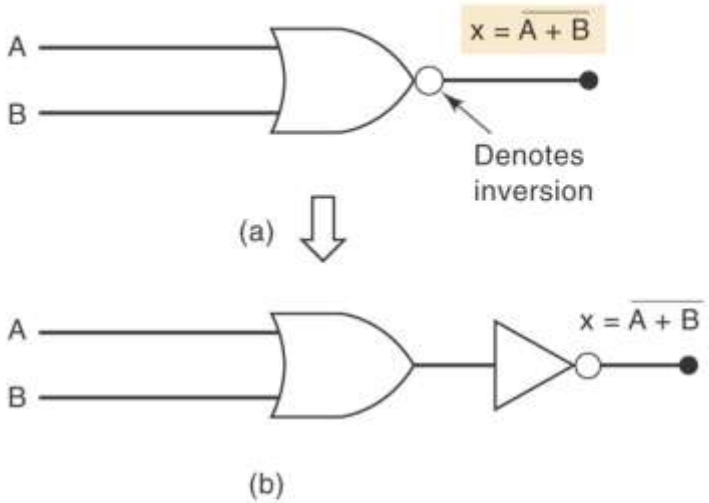


		AND	NAND
A	B	AB	$\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

(c)

NOR Gate

- Boolean expression for the NOR operation:
  $x = \overline{A + B}$

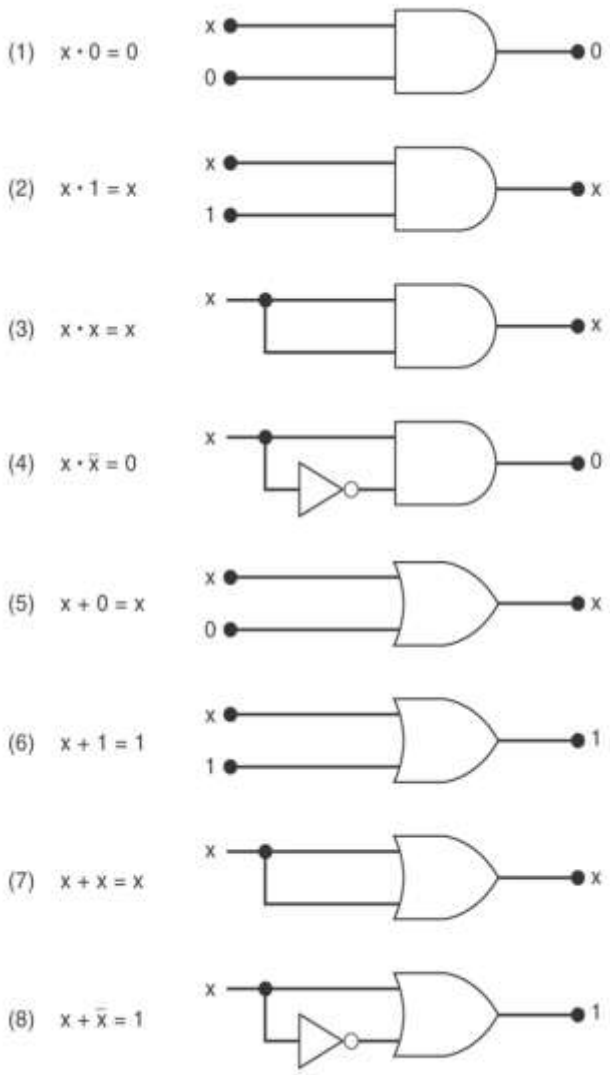


		OR	NOR
A	B	A + B	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

(c)

Boolean Theorems and Laws

- x* 0 =0 Fallacy
- x+1=1 Tautology
- x*x=x Indempotence Law
- x+x=x Indempotence Law
- x*x'=0 Complementarity Law
- x+x'=1 Complementarity Law
- x" = x Involution Law
- x+0=x
- x* 1 =x



Boolean Theorems and Laws

- x+y = y+x // Commutative Law
- x*y = y*x
- x+(y+z) = (x+y)+z=x+y+z // Associative Law
- x(yz)=(xy)z=xyz
- x(y+z)=xy+xz // Distributive Law
- (w+x)(y+z)=wy+xy+wz+xz
- x+xy=x // Absorption Law

Prove it by Truth Table

De'Morgan's Theorems

(Break the line change the sign)

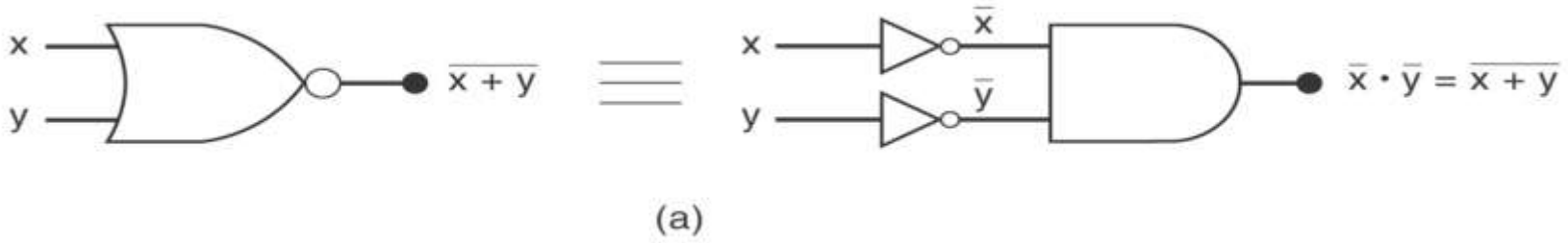
- (x+y)'=x'y'
- Implications and alternative symbol for NOR function
- (xy)'=x'+y'
- Implications and alternative symbol for NAND function
- Process of Demorganization:

  (i) Complement entire function

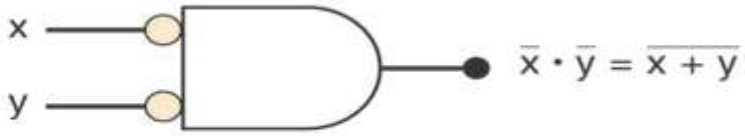
  (ii) Change all AND to OR and all OR to AND

  (iii) Complement each of the individual variables

Demorgan's Theorum

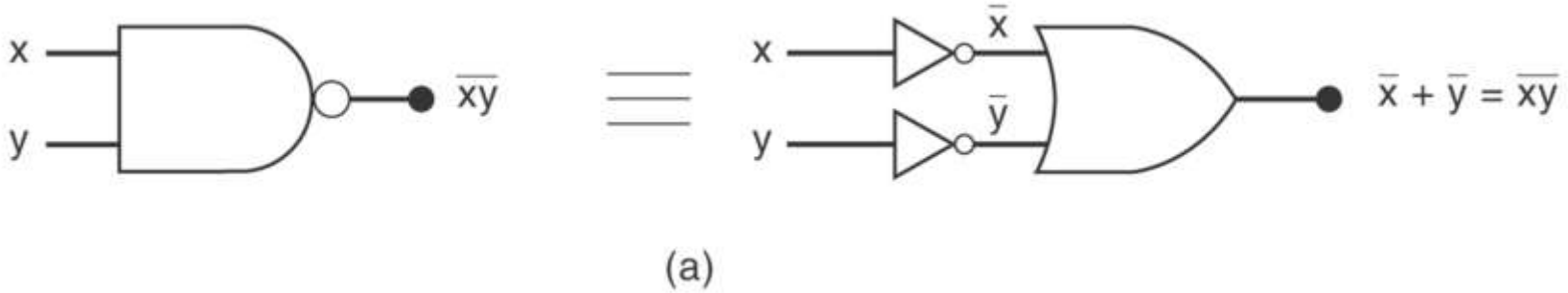


(a)

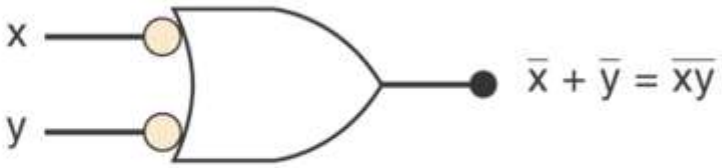


(b)

Demorgan's Theorum

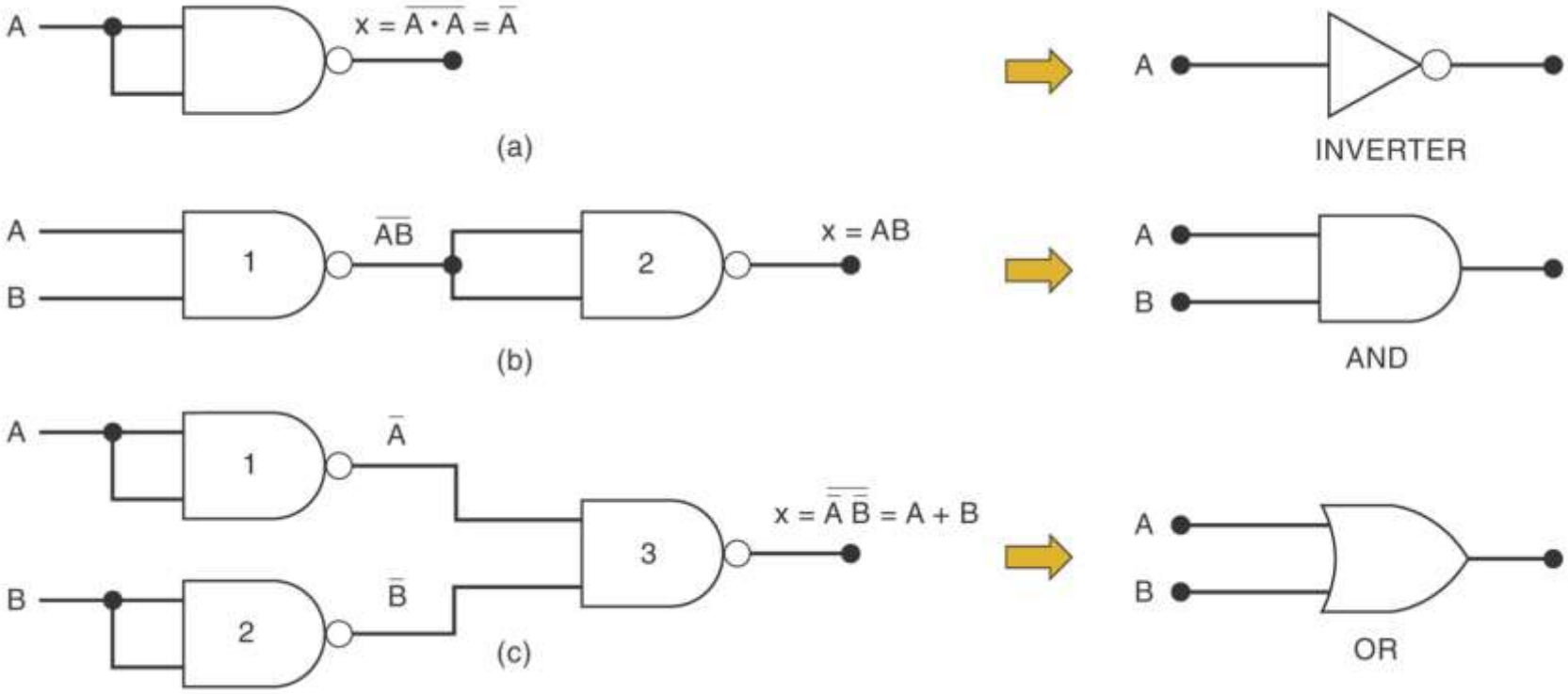


(a)



(b)

Universality of NAND Gates



Universality of NOR Gates